

## ISOTOPES

Objective: Understand what isotopes are
Be able to determine the number of protons and neutrons in an isotope given the mass number (A)
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It is the atomic number $(Z)$, or the number of protons, that defines an element. In other words, any atom of carbon ( $Z=$ 6) will always have 6 protons in its nucleus. But what about the number of neutrons?

It turns out that it is possible for different atoms of the same element to have different numbers of neutrons, and therefore different mass numbers ( $A$ ), which is the sum of the numbers of protons and neutrons. Such atoms are called isotopes.

We often use either of the following notations to represent atoms and isotopes of an element $(X)$ :

$$
{ }_{Z}^{A} X \quad \text { or } \quad{ }^{A} X
$$

As an example, let us consider the two isotopes of carbon shown below: carbon-12 $\left({ }^{12} \mathrm{C}\right)$ and carbon-13 $\left({ }^{13} \mathrm{C}\right)$.


The difference between these two atoms is that ${ }^{13} \mathrm{C}$ has one more neutron (gray sphere in the inset) ${ }^{12} \mathrm{C}$.

## AVERAGE ATOMIC MASS

Objective: How to calculate the average atomic mass of an element given the masses and abundances of its isotopes


Any natural sample of carbon will contain a mixture of the two stable isotopes, ${ }^{12} \mathrm{C}$ and ${ }^{13} \mathrm{C}$. Because the two isotopes have different mass numbers (A), due to different numbers of neutrons, they also have different masses (usually measured in units of amu).

As a result, the mass of natural carbon samples will always be comprised of these two isotopes. However, the two isotopes exist with different natural abundances in such samples, as shown in the table below.

| Isotope | Z | A | Mass (amu) | Abundance |
| :---: | :---: | :---: | :---: | :---: |
| ${ }^{12} \mathrm{C}$ | 6 | 12 | 12.000000 | $98.89 \%$ |
| ${ }^{13} \mathrm{C}$ | 6 | 13 | 13.003355 | $1.11 \%$ |



The mass presented on the periodic table for each element is actually the average atomic mass $\left(m_{\mathrm{X}}\right)$, which is the sum of the masses of each stable isotope ( $m_{1}, m_{2}, m_{3}, \ldots$ ), weighted by the natural abundances ( $a_{1}, a_{2}, a_{3}, \ldots$ ) of each isotope.

$$
m_{\mathrm{X}}=a_{1} m_{1}+a_{2} m_{2}+a_{3} m_{3}+\cdots
$$

For carbon, the average atomic mass works out to be:

$$
\begin{aligned}
m_{\mathrm{C}} & =(0.9889)(12.000000 \mathrm{amu})+(0.0111)(13.003355 \mathrm{amu}) \\
& =12.01 \mathrm{amu}
\end{aligned}
$$

## IONS

Objective: Understand what ions (cations and anions) are Be able to determine the number of electrons in ions
$\qquad$
In addition to having different numbers of neutrons, it is also possible for atoms of the same element to have different numbers of electrons. In these atoms, there is a net positive or net negative charge due to the unequal numbers of protons and electrons. These charged atoms are called ions.


Metals have a tendency to lose electrons to form positively charged ions called cations. Nonmetals have a tendency to gain electrons to form negatively charged ions called anions.

Some properties of ions, relative to neutral atoms, are also changed. For instance, the gain or loss of an electron will significantly increase or decrease the size of the ion, relative to the neutral atom, because of the uneven numbers of protons and electrons.


## PRACTICE PROBLEMS

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## Fundamental Concepts

1. Complete the following chart.

| Symbol: | ${ }_{30}^{64} \mathrm{Zn}$ | ${ }_{16}^{32} \mathrm{~S}$ | ${ }_{16}^{32} \mathrm{~S}^{2-}$ |
| :---: | :---: | :---: | :---: |
| \# protons |  |  |  |
| \# neutrons |  |  |  |
| \# electrons |  |  |  |
| Mass Number (A) |  |  |  |

2. What is the symbol for the element with a -1 charge, 36 electrons, and 46 neutrons?
3. Boron exists in two stable isotopes: ${ }^{10} \mathrm{~B}(10.0129 \mathrm{amu})$ and ${ }^{11} \mathrm{~B}$ ( 11.00931 amu ). If the atomic mass of boron is 10.811 amu , which is the best estimate for the natural abundances of each isotope?
a) $40 \%{ }^{10} \mathrm{~B}$ and $60 \%{ }^{11} \mathrm{~B}$
b) $20 \%{ }^{10} \mathrm{~B}$ and $80 \%{ }^{11} \mathrm{~B}$
c) $60 \%{ }^{10} \mathrm{~B}$ and $40 \%{ }^{11} \mathrm{~B}$

## Problem-Solving Skills

4. There are three stable isotopes of magnesium, with masses of $23.9850,24.9858$, and 25.9826 amu . If the average atomic mass of magnesium is 24.3050 amu and the natural abundance of the lightest isotope is $78.99 \%$, what are the natural abundances of the other two isotopes?
5. A compound $X C l_{2}$ contains an unknown ion ( $X^{n+}$ ). If the ion $X^{n+}$ contains 10 electrons, what is the identity of $X$ ?
6. A compound contains three times as many atoms of bromine as an unknown metal M. Metal M contains 23 electrons and 31 neutrons when it forms the compound. What is the mass number of $M$ ?
7. Consider the following two isotopes: ${ }^{18} \mathrm{~F}^{-}$and ${ }^{15} \mathrm{~N}^{3-}$.
a) Which isotopes have the same number of electrons as ${ }^{17} \mathrm{O}^{2-}$ ? There may be more than one answer.
b) Which isotopes have the same number of neutrons as ${ }^{17} \mathrm{O}^{2-}$ ? There may be more than one answer.

## PROBLEM 1

Complete the following chart. See chart below.

- answer -

Objective: Be able to determine the number of protons and neutrons in an isotope given atomic symbol Be able to determine the number of electrons in an ion

| Symbol: | ${ }_{30}^{64} \mathrm{Zn}$ | ${ }_{16}^{32} \mathrm{~S}$ | ${ }_{16}^{32} \mathrm{~S}^{2-}$ |
| :---: | :---: | :---: | :---: |
| \# protons | 30 | 16 | 16 |
| \# neutrons | 34 | 16 | 16 |
| \# electrons | 30 | 16 | 18 |
| Mass Number (A) | 64 | 32 | 32 |

Recognize that the atomic symbol ( $\left.{ }_{Z}^{A} \mathrm{X}\right)$ gives us the mass number $(A)$ as well as the atomic number ( $Z$ ).

For ${ }_{30}^{64} \mathrm{Zn}$, the atomic number is $\mathrm{Z}=30$, which means that there are 30 protons in the nucleus. There are also 30 electrons in the neutral atom since the charges from the protons ( +1 each) must cancel out with the charges from the electrons ( -1 each). Because the mass number ( $A=64$ ) is the sum of the number of protons and neutrons, we can determine there are 34 neutrons in the nucleus $(64=30+n)$.

For ${ }_{16}^{32} \mathrm{~S}^{2-}$, the atomic number is $\mathrm{Z}=16$, which means that there are 16 protons in the nucleus. There would be 16 electrons in the neutral atom, but since we have $\mathrm{S}^{2-}$ there are two extra electrons, so 18 electrons in total. Because the mass number $(\mathrm{A}=32)$ is the sum of the number of protons and neutrons, we can determine there are 16 neutrons in the nucleus $(32=16+n)$.

## PROBLEM 2

What is the symbol for the element with a -1 charge, 36 electrons, and 46 neutrons?

- answer -

Objective: Be able to determine the number of protons and neutrons in an isotope given atomic symbol
Be able to determine the number of electrons in an ion
First, recognize that we have an ion because there is an overall (net) negative ( -1 ) charge on the unknown atom.

Because we know the anion has 36 electrons, the neutral atom must have started with 35 electrons to begin with. Remember that electrons are negatively charged.

Second, if the neutral atom had 35 electrons, then it must also have 35 protons in order to balance out the charges. Remember that protons are positively charged. This means that the atomic number is $Z=35$, which is Br .

Third, the mass number $(\mathrm{A})$ is equal to the sum of the number of protons and neutrons: $A=35+46=81$.

Finally, putting all this information together into a symbol: ${ }_{35}^{81} \mathrm{Br}^{-}$

## PROBLEM 3

Boron exists in two stable isotopes: ${ }^{10} \mathrm{~B}(10.0129 \mathrm{amu})$ and ${ }^{11} \mathrm{~B}(11.00931 \mathrm{amu})$. If the atomic mass of boron is 10.811 amu , which is the best estimate for the natural abundances of each isotope?

- answer -

Objective: Be able to determine the average atomic mass of an element from isotope masses and natural abundances

Recall that the average atomic mass of any element is a weighted average of the stable isotopes of that element. This means that for boron, the average atomic mass can be determined from the following expression.

$$
m_{\mathrm{B}}=a_{1} m_{1}+a_{2} m_{2}
$$

where $m_{1}=10.0129 \mathrm{amu}, m_{2}=11.00931 \mathrm{amu}$, and $a_{1}$ and $a_{2}$ are the natural abundances of the two isotopes, respectively.

Because the average atomic mass is 10.811 amu , we can expect the heavier isotope $\left({ }^{11} \mathrm{~B}\right)$ to have a greater abundance than the lighter isotope $\left({ }^{(10} \mathrm{B}\right)$. In other words: $a_{2}>a_{1}$. This leaves us with choices ( a ) and ( b ).

Moreover, notice that the average atomic mass is slightly closer in value to the mass of ${ }^{11} \mathrm{~B}$ isotope's mass. Choice (a) is a bit more evenly split, which would result in an average atomic mass closer to 10.5 amu . Choice (b) is more abundant in ${ }^{11} \mathrm{~B}$, so that is the best estimate.

Note: You could also plug int the percentages to see which gives the closest answer.

## PR○BLEM 4

There are three stable isotopes of magnesium, with masses of 23.9850 , 24.9858 , and 25.9826 amu . If the average atomic mass of magnesium is 24.3050 amu and the natural abundance of the lightest isotope is $78.99 \%$, what are the natural abundances of the other two isotopes?

## - answer -

## Objective: Be able to determine the natural abundances of isotopes given their masses and the average atomic mass

Recall that the average atomic mass of any element is a weighted average of the stable isotopes of that element. For magnesium, this is:

$$
m_{\mathrm{Mg}}=a_{1} m_{1}+a_{2} m_{2}+a_{3} m_{3}
$$

where $m_{1}=23.9850 \mathrm{amu}, m_{2}=24.9858 \mathrm{amu}, m_{3}=25.9826 \mathrm{amu}$, and $a_{1}=0.7899$. We want to determine $a_{2}$ and $a_{3}$. Let's reassign $a_{2}=x$. Because the abundances must sum to $100 \%$, we can formulate the following expressions for $a_{2}$ and $a_{3}$ :

$$
a_{1}+a_{2}+a_{3}=1.0 \rightarrow a_{1}=0.7899 \quad a_{2}=x \quad a_{3}=1.0-0.7899-x=0.2101-x
$$

We can use these expressions for $a_{2}$ and $a_{3}$ to plug into the average atomic mass formula above to solve for $x$ (which is $a_{2}$ ):

$$
\begin{aligned}
m_{\mathrm{Mg}} & =a_{1} m_{1}+x m_{2}+(0.2101-x) m_{3} \\
24.3050 \mathrm{amu} & =(0.7899)(23.9850 \mathrm{amu})+(x)(24.9858 \mathrm{amu})+(0.2101-x)(25.9826 \mathrm{amu}) \\
x & =0.1000=a_{2}
\end{aligned}
$$

Therefore, the abundances are:

$$
a_{1}=0.7899 \quad a_{2}=0.1000 \quad a_{3}=0.1101 \quad \rightarrow \quad a_{1}=78.99 \% \quad a_{2}=10.00 \% \quad a_{3}=11.01 \%
$$

## PROBLEM 5

A compound $\mathrm{XCl}_{2}$ contains an unknown ion ( $\mathrm{X}^{\mathrm{n}+}$ ). If the ion $\mathrm{X}^{\mathrm{n+}}$ contains 10 electrons, what is the identity of X ?

- answer -


## Objective: Be able to determine the charge of an ion

Be able to determine the identity of an atom from its atomic number ( ( )
First, recognize that the compound contains chloride anions: $\mathrm{Cl}^{-}$.

Because there are two chloride anions in the compound, the charge of the X cation must be +2 or $\mathrm{X}^{2+}$ to make the compound neutral overall.

Second, if $\mathrm{X}^{2+}$ cation contains a charge of +2 , then the cation had to lose two electrons because each electron carries a -1 charge. Therefore, the neutral atom of $X$ would have contained 12 electrons originally.

Third, the neutral atom would need to have 12 protons, or an atomic number of $Z=12$, which corresponds to Mg .

The identity of X is Mg ( or $\mathrm{Mg}^{2+}$ ).

## PROBLEM 6

A compound contains three times as many atoms of bromine as an unknown metal $M$. Metal $M$ contains 23 electrons and 31 neutrons when it forms the compound. What is the mass number of M ?

- ans\%er -

Objective: Be able to determine the charge of an ion
Be able to determine the identity of an atom from its atomic number ( $\mathcal{Z}$ )
First, recognize that the compound has this general formula: $\mathrm{MBr}_{3}$.

Because there are three bromide anions ( $\mathrm{Br}^{-}$) in the compound, the charge of the M cation must be +3 or $\mathrm{M}^{3+}$ to make the compound neutral overall.

Second, if $\mathrm{M}^{3+}$ cation contains a charge of +3 , then the cation had to lose three electrons because each electron carries a -1 charge. Therefore, the neutral atom of $M$ would have contained 26 electrons originally.

Third, the neutral atom would need to have 26 protons, or an atomic number of $Z=26$, which corresponds to Fe.

The mass number for Fe would be $\mathrm{A}=26+31=57$.

## PROBLEM 7

Consider the following two isotopes: ${ }^{18} \mathrm{~F}^{-}$and ${ }^{15} \mathrm{~N}^{3-}$.
a) Which isotopes have the same number of electrons as ${ }^{17} \mathrm{O}^{2-}$ ? There may be more than one answer.
b) Which isotopes have the same number of neutrons as ${ }^{17} \mathrm{O}^{2-}$ ? There may be more than one answer.

- answer -

Objective: Be able to determine the number of protons and neutrons in an isotope
Be able to determine the number of electrons in an ion
For each ion, let's determine the number of protons, neutrons, and electrons.

| Symbol: | ${ }_{9}^{18} \mathrm{~F}^{-}$ | ${ }_{7}^{15} \mathrm{~N}^{3-}$ | ${ }_{8}^{17} \mathrm{O}^{2-}$ |
| :---: | :---: | :---: | :---: |
| \# protons | 9 | 7 | 8 |
| \# neutrons | 9 | 8 | 9 |
| \# electrons | 10 | 10 | 10 |
| Mass Number (A) | 18 | 15 | 17 |

Therefore, both ${ }_{9}^{18} \mathrm{~F}^{-}$and ${ }_{7}^{15} \mathrm{~N}^{3-}$ contains the same number of electrons (10) as ${ }_{8}^{17} \mathrm{O}^{2-}$.

And, ${ }_{9}^{18} \mathrm{~F}^{-}$contain the same number of neutrons (9) as ${ }_{8}^{17} \mathrm{O}^{2-}$.

